



1 Technical Note: An efficient method for accelerating the spin-up
2 process for process-based biogeochemistry models

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15 **Abstract**

16 To better understand the role of terrestrial ecosystems in the global carbon cycle and their
17 feedbacks to the global climate system, process-based biogeochemistry models need to be
18 improved with respect to model parameterization and model structure. To achieve these
19 improvements, the spin-up time for those differential equation-based models needs to be
20 shortened. Here, an algorithm for a fast spin-up was developed and implemented in a
21 biogeochemistry model, the Terrestrial Ecosystem Model (TEM). With the new spin-up
22 algorithm, we showed that the model reached a steady state in less than 10 years of computing
23 time, while the original method requires more than 200 years on average of model run. For the
24 test sites with five different plant function types, the new method saves over 90% of the original
25 spin-up time in site-level simulations. In North America simulations, average spin-up time
26 saving for all grid cells is 85% for either daily or monthly version of TEM. The developed spin-
27 up method shall greatly facilitate our future quantification of carbon dynamics at fine spatial and
28 temporal scales.

29



30 1. Introduction

31 Biogeochemistry models contain state variables representing various pools of carbon and
32 nitrogen and a set of flux variables representing the element and material transfers between
33 different state variables. Model spin-up is a step to get biogeochemistry models to a steady state
34 for those state and flux variables (McGuire et al., 1992; King, 1995; Johns et al., 1997;
35 Dickinson et al., 1998). Spin-up normally uses cyclic forcing data to force the model run, and
36 reach a steady state, which will be used as initial conditions for model transient simulations. The
37 steady state is reached when modeled state variables show a cyclic pattern or a constant and
38 often requires a significant amount of computation time, which needs to be accelerated for
39 regional and global simulations at fine spatial and temporal scales.

40 Spin-up is normally achieved by running model repeatedly using one or several decades of
41 meteorological or climatic data, until a steady state is reached. The step could require model
42 repeatedly run for more than 2000 annual cycles. Specifically, the model will check the stability
43 of the simulated carbon and nitrogen fluxes as well as state variables with specified threshold
44 values. For instance, the model will check if the simulated annual net ecosystem production
45 (NEP) is less than $1 \text{ g C m}^{-2} \text{ yr}^{-1}$ (McGuire et al., 1992). Another method to reach a steady state
46 is to obtain the analytical solutions (King et al, 1995; Comins, 1997), which might also take a
47 significantly long time.

48 For different biogeochemistry models, spin-up could take hundreds and thousands of years to
49 reach a stability, normally longer than the model projection period (Thornton et al., 2005).
50 Therefore, a more efficient method to reach the steady state will speed up the entire model
51 simulation. Recently, a semi-analytical method (Xia et al., 2012) has been adapted to a carbon-
52 nitrogen coupled model to speed up the spin-up process. The idea is to get an analytical solution



53 very close to a steady condition, then start spin-up from the solution, which could significantly
54 reduce spin-up time. However, this technique did not reach a cyclic pattern for state and flux
55 variables and required an additional spin-up process to achieve the steady state.

56 Here we developed a new method to accelerate the spin-up process. We tested the method
57 for representative plant function types and the North America with both daily and monthly
58 versions of TEM (Zhuang et al., 2003). In addition, we compared the performance of our
59 algorithms with the semi-analytical version of Xia et al. (2012). The new algorithms shall help
60 us conduct very high spatial and temporal resolution simulations with process-based
61 biogeochemistry models in the future.

62

63 2. Method

64 2.1 TEM description

65 We used a process-based biogeochemistry model, the Terrestrial Ecosystem Model (TEM;
66 Zhuang et al. 2003) as testbed to demonstrate the performance of the new algorithms of spin-up.
67 TEM simulates the dynamics of ecosystem carbon and nitrogen fluxes and pools (McGuire et al.,
68 1992; Zhuang et al., 2010, 2003). It contains five state variables: carbon in living vegetation (C_v),
69 nitrogen in living vegetation (N_v), organic carbon in detritus and soils (C_s), organic nitrogen in
70 detritus and soils (N_s), and available inorganic soil nitrogen (N_{av}). Carbon and nitrogen
71 dynamics in TEM are governed by following equations:



$$\frac{dC_v}{dt} = GPP - R_A - L_C \dots\dots\dots(1)$$

$$\frac{dN_v}{dt} = NUPTAKE - L_N \dots\dots\dots(2)$$

$$\frac{dC_s}{dt} = L_c - R_H \dots\dots\dots(3)$$

$$\frac{dN_s}{dt} = L_N - NETNMIN \dots\dots\dots(4)$$

$$\frac{dN_{ev}}{dt} = NINPUT + NETNMIN - NLOST - NUPTAKE \dots\dots(5)$$

72

73 Where GPP is gross primary production, R_A is autotrophic respiration, L_C is carbon in
 74 litterfall, $NUPTAKE$ is nitrogen uptake by vegetation, L_N is nitrogen in litterfall, R_H is
 75 heterotrophic respiration, $NETNMIN$ is net rate of mineralization of soil nitrogen, $NINPUT$ is
 76 nitrogen input from outside ecosystem, $NLOST$ is nitrogen loss from ecosystem. Key carbon
 77 fluxes are defined as:

$$GPP = C_{max} \cdot f(PAR) \cdot f(PHENOLOGY) \cdot f(FOLIAGE) \cdot f(T) \cdot f(C_a, G_v) \cdot f(NA) \cdot f(FT) \dots\dots\dots(6)$$

$$NPP = GPP - R_A \dots\dots\dots(7)$$

$$NEP = GPP - R_A - R_H \dots\dots\dots(8)$$

78

79 NEP will be near zero when the ecosystem reaches a steady state. Therefore, the spin-up
 80 goal is to keep running the model driven with repeated climate forcing data until NEP is close to
 81 zero with a certain tolerance value (e.g., $0.1 \text{ g C m}^{-2} \text{ yr}^{-1}$).

82 2.2 Spin-up acceleration method

83 TEM can be re-formulated as:

$$\frac{d\bar{x}}{dt} = g(\bar{x}) + \bar{h} \dots\dots\dots(9)$$

84

85



86 Where x is a vector of state variables (e.g., V_c); \vec{h} is vector of carbon/nitrogen input from the
 87 atmosphere, independent on \vec{x} ; $g(x)$ is the process rate function of element pools (e.g., GPP).

88 By linearizing the model in term of pools, we could get:

89
$$g(\vec{x}, t) = g(\vec{x}_0, t) + J(\vec{x} - \vec{x}_0) \dots \dots \dots (10)$$

90 Where J is the Jacobian matrix of the process rate:

91
$$J = \frac{dg}{dx} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdot & \cdot & \cdot & \frac{\partial g_1}{\partial x_n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \frac{\partial g_n}{\partial x_1} & \cdot & \cdot & \cdot & \frac{\partial g_n}{\partial x_n} \end{bmatrix} \dots \dots \dots (11)$$

92 The numerical discretization of equation (9) is:

93
$$x_k - x_{k-1} = \tau \cdot J_{k-\frac{1}{2}} \cdot x_{k-1} + \tau \left(g(x_{0,k-1}) - J \cdot x_{0,k-1} + h_{k-1} \right) \dots \dots \dots (12)$$

94 Where τ is time step (month), x_k is pool size at time k , $J_{k-\frac{1}{2}}$ is a Jacobian matrix at time step

95 $k - \frac{1}{2}$ (half timestep).

96 We introduce:

97
$$f_{k-1} = g(x_{0,k-1}) - J \cdot x_{0,k-1} + h_{k-1} \dots \dots \dots (13)$$

98 The equation can then be written as:



99
$$x_k - x_{k-1} = \tau \cdot J_{k-\frac{1}{2}} \cdot x_{k-1} + \tau \cdot f_{k-1} \dots \dots \dots (14)$$

100 Where $J_{k-\frac{1}{2}}$ is a Jacobian matrix at time step $k - \frac{1}{2}$ (half step). After running a large
 101 number of annual cycles, model approaches a cyclo-stationary state, which can be expressed by
 102 condition $x^{T+i} = x^i$, where T is the number of time steps in one cycle. For example, when spin
 103 up is made at monthly time step using monthly climatology of temperature, precipitation and
 104 other forcing data, T equals 12, and x^1 is the size of carbon pools on January 1st, while $J^{1.5}$ is
 105 the matrix of mean process rate constants for January.

106 By introducing

107
$$A_k = \tau \cdot J_{k-\frac{1}{2}}, y_k = \tau f_{k-\frac{1}{2}}, B_k = I, C_k = I + A$$

108 where I is an identity matrix.

109 Eq. (12) can be written as:

110
$$-C_k \cdot x_{k-1} + B_k \cdot x_k = y_k \dots \dots \dots (15)$$

111 The cyclic boundary condition is: $x_1 = x_{T+1}$

112 Then Eq. (13) will become:

113
$$-C_1 \cdot x_T + B_1 \cdot x_1 = y_1 \dots \dots \dots (15a)$$

114 Thus equations (15, 15a) become a formulation of a linear problem with T unknown vectors
 115 x_k , which can be solved using LU decomposition or Gaussian elimination. Xia et al (2012, see



116 Eq. 4) and Kwon and Primeau (2006) also had linear equations for a steady state, but only for
 117 annually averaged mean value. Going for annual average form reduces the size of problem, but
 118 prevents Xia et al (2012) from obtaining exact solution of the system (see their Eq. 3, 3a),
 119 because introducing cyclic boundary conditions in their Eq. (3a) was missing in their methods.

120 2.2 Numerical Implementation

121 Equation (15a) is explicitly expressed as:

122

$$\begin{pmatrix} B & 0 & 0 & 0 & 0 & 0 & -C \\ -C & B & 0 & 0 & 0 & 0 & 0 \\ 0 & -C & B & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & -C & B & 0 & 0 \\ 0 & 0 & 0 & 0 & -C & B & 0 \\ 0 & 0 & 0 & 0 & 0 & -C & B \end{pmatrix} \times \begin{pmatrix} x^1 \\ x^2 \\ \dots \\ x^k \\ \dots \\ x^f \end{pmatrix} = \begin{pmatrix} y^1 \\ \dots \\ y^f \end{pmatrix} \dots\dots\dots(16)$$

123 Eq. (16) can be shown in form $Mx = Y$.

124 Apply the Gaussian elimination to upper block that reduces M to a lower triangular form and
 125 the elimination process is applied from right to left in the top row of M involving 2x2 blocks of
 126 matrices B, C, D and D'.

127

$$\begin{pmatrix} D^1 & D \\ -C_k & B_k \end{pmatrix} \begin{pmatrix} y_1 \\ y_k \end{pmatrix} \dots\dots\dots(17)$$

128

129 The result matrix is:



$$M' = \begin{pmatrix} B' & 0 & 0 & 0 & 0 & 0 & 0 \\ -C & B & 0 & 0 & 0 & 0 & 0 \\ 0 & -C & B & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & -C & B & 0 & 0 \\ 0 & 0 & 0 & 0 & -C & B & 0 \\ 0 & 0 & 0 & 0 & 0 & -C & B \end{pmatrix} \dots\dots\dots(18)$$

130

131 The solution of eq. (15a) will be readily obtained for x.

132 2.3 Algorithm implementation to TEM

133 In original TEM, carbon fluxes can be defined as:

$$NPP = GPP - MR - GR \dots\dots\dots(19)$$

134 $MR = V_C \cdot K_T \dots\dots\dots(20)$

$$GR = \begin{cases} 0.25 \cdot (GPP - MR), & \text{if } GPP > MR \\ 0 & , \text{ otherwise} \end{cases} \dots\dots\dots(21)$$

135 Where net primary production (NPP) is defined as the difference of GPP and plant maintenance
 136 respiration (MR) and growth respiration (GR). MR is assumed as a function of V_C and
 137 temperature (K_T). Here we revised MR calculation:

138 $MR = \begin{cases} V_C \cdot K_T, & \text{if } GPP > V_C \cdot K_T \\ 0.75 \cdot V_C \cdot K_T + 0.25 \cdot GPP, & \text{otherwise} \end{cases} \dots\dots\dots(21)$

139 The net ecosystem production (NEP) is defined as the difference between NPP and
 140 heterotrophic respiration (R_H).

141 The basic workflow to implement the method is: 1) linearizing TEM first to get a sparse
 142 matrix with n-variable system; 2) performing Gaussian elimination for the linear system; 3)
 143 solving the sparse matrix to acquire the state variable values (Figure 1). To adapt this method to



144 a daily version of TEM, we changed the cyclic condition T from 12 to 365. The other steps are
145 the same as monthly version. We tested the new method for carbon only version and carbon-
146 nitrogen coupled version of TEM for different PFTs (Table 1). Specifically, for the carbon only
147 version, we only solved the differential equations that govern the carbon dynamics, while for the
148 carbon-nitrogen coupled version, we solved the differential equations that govern both carbon
149 and nitrogen dynamics in the system. For the both versions, the spin-up process strives to reach
150 a steady state for carbon pools and fluxes.

151 3. Results and Discussion

152 At Harvard Forest site, the traditional spin-up method took 564 years to get the steady state
153 for both the carbon-only and coupled carbon–nitrogen simulations with annual NEP less than 0.1
154 $\text{g C m}^{-2} \text{yr}^{-1}$ (Figure 2). The improved method took 72 years for the carbon only and 122 for the
155 coupled carbon–nitrogen simulations, respectively. For carbon and nitrogen pools, it took
156 another 45 years (equivalent cyclic time) to reach a steady state with NEP less than 0.1 g C m^{-2}
157 yr^{-1} . In comparison with the traditional spin-up method (Zhuang et al., 2003), the new method
158 saved 65% of computational time to get the steady state in the carbon-only simulations (Table 2).
159 The differences in steady-state carbon pools between using the new method and traditional spin-
160 up methods were small (less than 0.85%). Similarly, for the coupled carbon–nitrogen
161 simulations, the new method saves a similar amount of time to reach the steady state. For the
162 seven test sites, it takes on average 0.6 seconds using new method to reach a steady state.
163 Compared to the original spin-up method, the new method is not only faster, but also
164 computationally stable.

165 The time of spin-up to reach a steady state of NEP varied for different PFT grids using the
166 original method (Figure 2). In general, to allow 98% grid cells reach their steady states of NEP,



167 it will take 250 annual model runs. While the new method will only need on average 0.6 seconds
168 (equivalent to 60-year annual model runs with the original method) (Figure 3). For regional tests
169 in North America, we found that the average saving time with the new method with monthly
170 TEM is 25%, 32%, and 22%, for Alaska, Canada, and the conterminous US, respectively.

171

172 To compare the performance of the new method with other existing methods, we adapted the
173 semi-analytical method (Xia et al., 2012) to TEM model. To do that, we first revised the TEM
174 model structure to:

175
$$\frac{dP(t)}{dt} = \varepsilon ACP(t) \dots \dots \dots (22)$$

176 Where $P(t)$ is a vector of pools in TEM (e.g., V_C and S_C). ε is a scalar. A is a pool transfer matrix
177 (in which A_{ij} represents the fraction of carbon transfer from pool j to i). C is a diagonal matrix
178 with pool components (where diagonal components quantify the fraction of carbon left from the
179 state variables after each time step). With this method, we obtained an analytical solution for the
180 intermediate state. We then kept running TEM with the traditional spin-up process. Specifically,
181 we started TEM simulation to estimate the state variable values. Based on these values, the spin-
182 up runs were conducted to reach the final steady state. We found that the semi-analytical solution
183 is better than the original spin-up method, but slower than the new method proposed in this study
184 (Table 2).

185 4. Summary

186 We developed a new method to speed up the spin-up process in process-based
187 biogeochemistry models. We found that the new method shortened 90% of the spin-up time



188 using the traditional method. For regional simulations in North America, average spin-up time
189 saving is 85% for either daily or monthly version of TEM. This method will significantly help
190 our future carbon dynamics quantification with biogeochemistry models at fine spatial and
191 temporal scales.

192



193 Table 1. Test sites for new spin-up algorithms

Site Name	Location	PFT	Reference
1. Fort Peck	48.3N, 105.1W	Grassland	Gilmanov et al. [2005]
2. Bartlett Exp Forest	44.1N, 71.3W	Deciduous broadleaf	Ollinger et al. [2005]
3. UCI_1850	55.9N, 98.5W	Evergreenn needle-leaf	Goulden et al. [2006]
4. Vaira Ranch	38.4N, 121.0W	Grassland	Baldocchi et al. [2004]
5. Missouri Ozark	38.7N, 92.2	Deciduous broadleaf	Gu et al. [2007, 2012]
6. Niwot Ridge	40.0N, 105.5W	Evergreenn needle-leaf	Turnipseed et al. [2003, 2004]
7. Harvard Forest	43.5N, 72.2W	Deciduous broadleaf	Van Gorsel et al. [2009]

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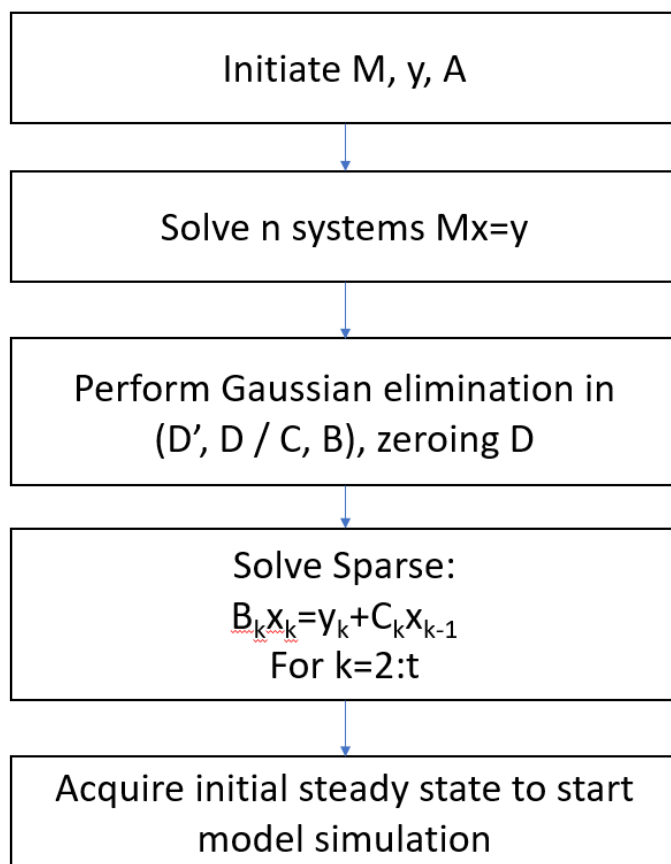
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206 Table 2. Spin-up time comparison for different methods, seconds represent real computation time,
207 years refer to the spin-up annual cycles

Site No.	Original Spin-up Year	Spin-up computation time (Seconds)	New method computation time (Seconds)	Semi-analytical method (equivalent annual cycles)
1	231	1.3	0.5	0.7s (+76)
2	305	1.7	0.3	0.8s (+101)
3	245	1.5	0.4	0.9s (+52)
4	443	2.2	0.4	0.5s (+118)
5	304	1.8	0.4	0.8s (+86)
6	204	1.1	0.3	0.7s (+43)
7	564	2.5	0.6	0.9(+45)

208



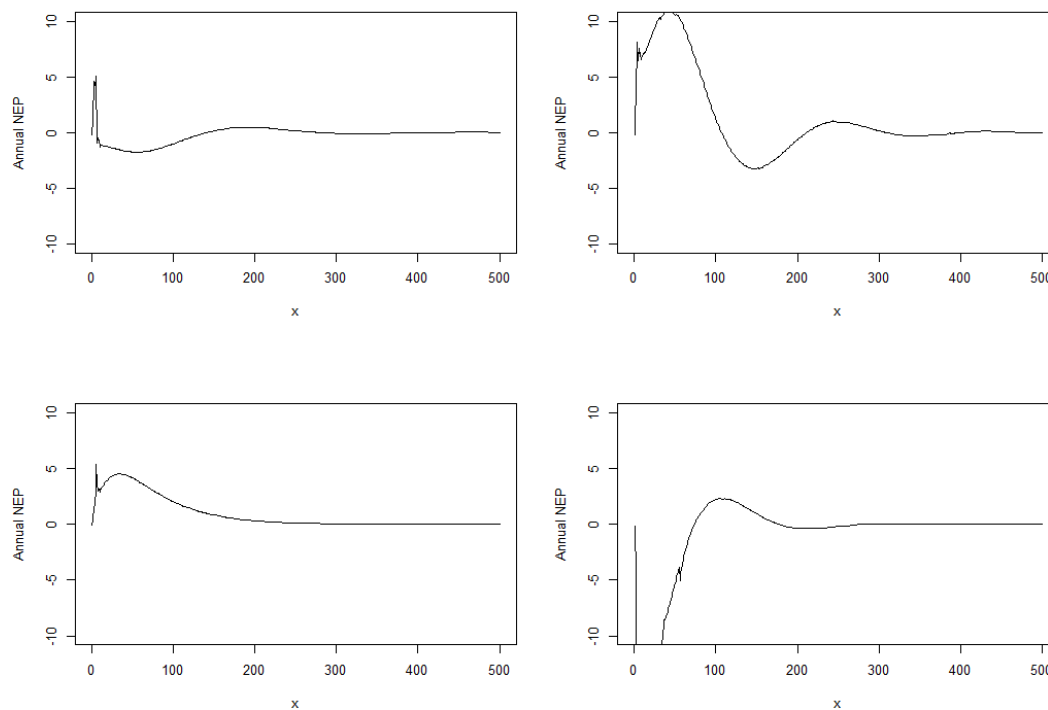
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210 Fig. 1. Algorithms and procedures of the new spin-up method

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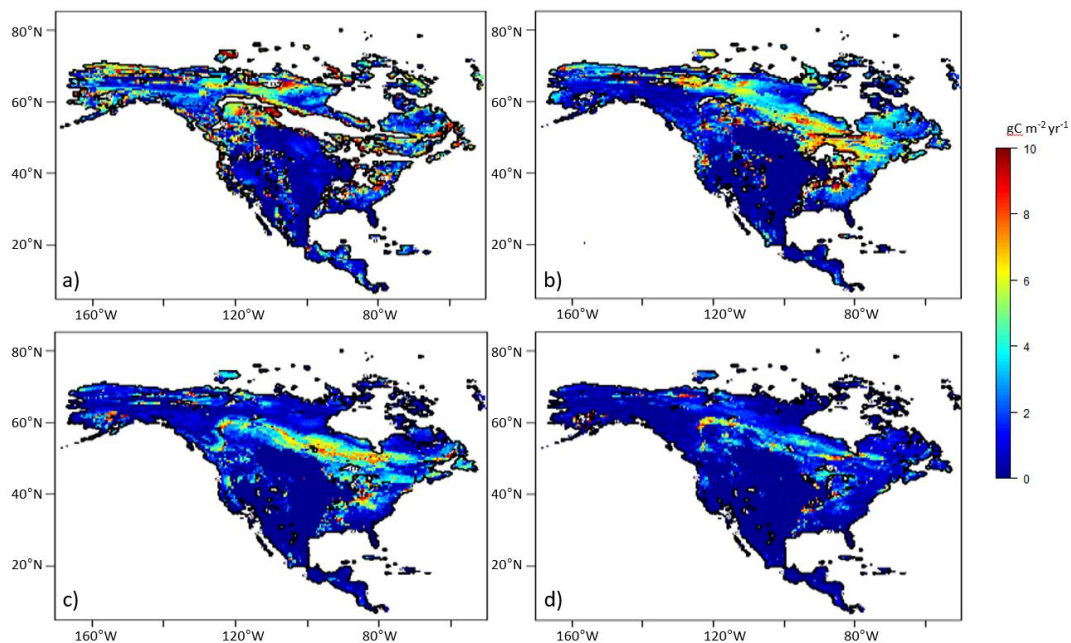
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215 Fig. 2. The time for NEP ($\text{g C yr}^{-1}\text{m}^{-2}$) reached a steady state with the original spin-up method at
216 Harvard forest site. x represents model simulation years.

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221 Fig. 3. The spin-up time to reach the steady state of NEP ($\text{g C m}^{-2} \text{yr}^{-1}$) with the original spin-
222 method: In 50, 100, 150, and 200 years, 63%, 89%, 93%, and 98% grids will reach their steady
223 states, respectively.

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